**1.** The heights of females from a country are normally distributed with

- a mean of 166.5 cm
- a standard deviation of 6.1 cm

Given that 1% of females from this country are shorter than $k$ cm,

(a) find the value of $k$

**(2)**

(b) Find the proportion of females from this country with heights between 150 cm and 175 cm

**(1)**

A female, from this country, is chosen at random from those with heights between 150 cm and 175 cm

(c) Find the probability that her height is more than 160 cm

**(4)**

The heights of females from a different country are normally distributed with a standard deviation of 7.4 cm

Mia believes that the mean height of females from this country is less than 166.5 cm

Mia takes a random sample of 50 females from this country and finds the mean of her sample is 164.6 cm

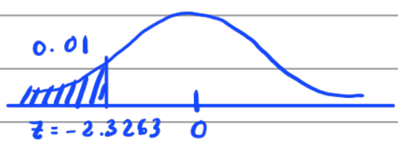
(d) Carry out a suitable test to assess Mia's belief.
You should

- state your hypotheses clearly
- use a 5% level of significance

**(4)**

a) let $F \sim N(166.5, 6.1^2)$ ← $X \sim N(\mu, \sigma^2)$ is a Normal Hypothesis Test
mean std

$P(F < K) = 0.01$ ← $0.01 = 1\%$

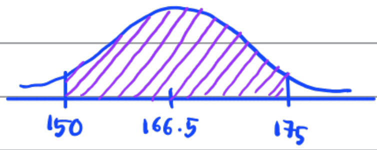

$\frac{x-\mu}{\sigma} = z$ : $\frac{K - 166.5}{6.1} = -2.3263$ (1)

$\therefore k = 152.309$

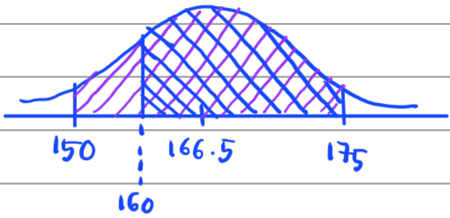
$= 152$ (3 s.f.) (1)

b) $P(150 < F < 175) = 0.91484\ldots$

$= 0.915 \ (3 \text{ s.f.})$ (1)




c)




$P(F > 160 \mid 150 < F < 175)$ (1) $= \dfrac{P(160 < F < 175)}{P(150 < F < 175)}$ (1)

$= \dfrac{0.7749487\ldots}{0.91484\ldots}$ (1)

$= 0.84708\ldots$

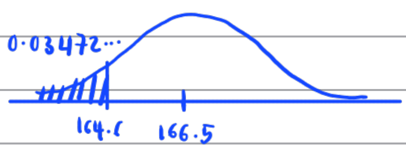
$= 0.847 \ (3 \text{ s.f.})$ (1)

d) Let $X$ = heights of females from 2nd country

$\bar{X} \sim N\left(166.5, \left(\frac{7.4}{\sqrt{50}}\right)^2\right)$ (1)

$H_0 : \mu = 166.5$, $H_1 : \mu < 166.5$ (1)

$P(\bar{X} < 164.6) = 0.03472\ldots$ (1) $< 0.05$ $\therefore$ So, reject $H_0$. (1)




There is evidence to support Mia's belief.

**2.** George throws a ball at a target 15 times.
Each time George throws the ball, the probability of the ball hitting the target is 0.48

The random variable $X$ represents the number of times George hits the target in 15 throws.

(a) Find

(i) $P(X = 3)$

(ii) $P(X \geqslant 5)$

**(3)**

George now throws the ball at the target 250 times.

(b) Use a normal approximation to calculate the probability that he will hit the target more than 110 times.

**(3)**

a) $X \sim B(15, 0.48)$ 1

(i) $P(X=3) = 0.019668...$
$= 0.0197$ (3sf) 1

(ii) $P(X \geq 5) = 1 - P(X \leq 4)$
$= 0.92013...$
$= 0.920$ (3sf) 1

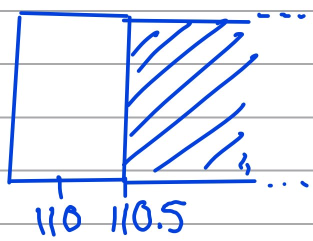
b) let Y be the number of times George hits the target.

$\mu = np = 250 \times 0.48 = 120$
$\sigma = \sqrt{np(1-p)} = \sqrt{62.4}$

$Y \sim N(120, \sqrt{62.4}^2)$ 1

$P(X > 110)$

$\approx P(Y > 110.5)$ 1




$= 0.88544...$
$= 0.885$ (3sf) 1

**3.** A study was made of adult men from region $A$ of a country.
It was found that their heights were normally distributed with a mean of 175.4 cm and standard deviation 6.8 cm.

(a) Find the proportion of these men that are taller than 180 cm.

**(1)**

A student claimed that the mean height of adult men from region $B$ of this country was different from the mean height of adult men from region $A$.

A random sample of 52 adult men from region $B$ had a mean height of 177.2 cm

The student assumed that the standard deviation of heights of adult men was 6.8 cm both for region $A$ and region $B$.

(b) Use a suitable test to assess the student's claim.
You should

- state your hypotheses clearly
- use a 5% level of significance

**(4)**

(c) Find the $p$-value for the test in part (b)

**(1)**

a) Let r.v. $X$ = height from region A

$X \sim N(175.4, 6.8^2)$

$P(X > 180) = 0.2493\ldots = 0.249$ (3 s.f.) ①

b) Let r.v. $Y$ = height from region B

$\bar{Y} \sim N\left(175.4, \frac{6.8^2}{52}\right)$ ①

$H_0 : \mu = 175.4$ , $H_1 : \mu \neq 175.4$ ①

$\therefore P(\bar{Y} > 177.2) = 0.0281\ldots > 0.025$. (two-tailed, so $\alpha = 0.025$) ①

$\therefore$ do not reject $H_0$ as there is insufficient evidence to support student's claim. ①

c) $p = 2 \times 0.0281\ldots$

$= 0.05628,\ldots$ (1)

**4.** A medical researcher is studying the number of hours, $T$, a patient stays in hospital following a particular operation.

The histogram on the page opposite summarises the results for a random sample of 90 patients.

(a) Use the histogram to estimate $P(10 < T < 30)$

**(2)**

For these 90 patients the time spent in hospital following the operation had

- a mean of 14.9 hours
- a standard deviation of 9.3 hours

Tomas suggests that $T$ can be modelled by $N(14.9, 9.3^2)$

(b) With reference to the histogram, state, giving a reason, whether or not Tomas' model could be suitable.

**(1)**

Xiang suggests that the frequency polygon based on this histogram could be modelled by a curve with equation

$$y = kxe^{-x} \quad 0 \leqslant x \leqslant 4$$

where

- $x$ is measured in **tens of hours**
- $k$ is a constant

(c) Use algebraic integration to show that

$$\int_0^n xe^{-x}\,dx = 1 - (n + 1)e^{-n}$$

**(4)**

(d) Show that, for Xiang's model, $k = 99$ to the nearest integer.

**(3)**

(e) Estimate $P(10 < T < 30)$ using

(i) Tomas' model of $T \sim N(14.9, 9.3^2)$

**(1)**

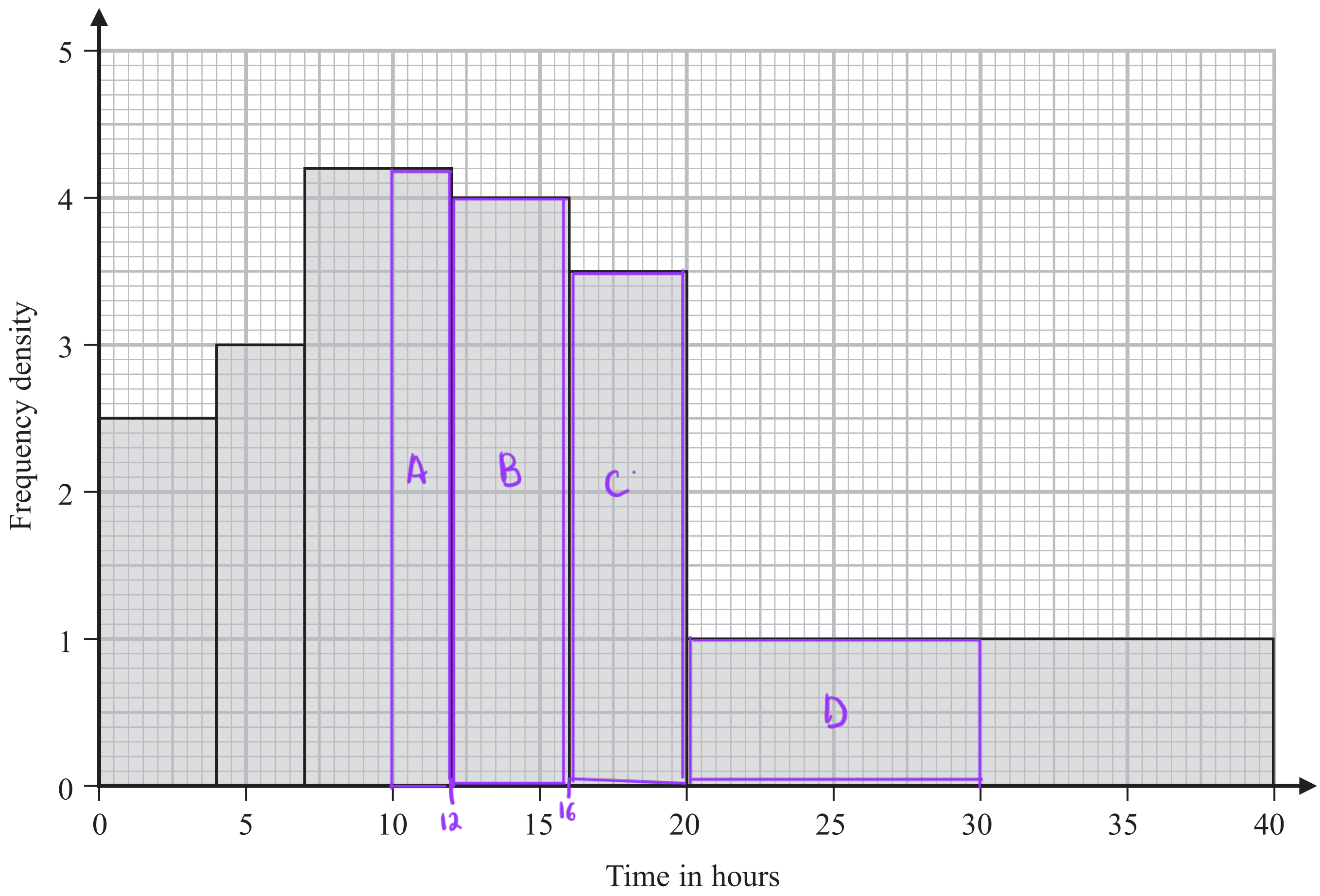
(ii) Xiang's curve with equation $y = 99xe^{-x}$ and the answer to part (c)

**(2)**

The researcher decides to use Xiang's curve to model $P(a < T < b)$

(f) State one limitation of Xiang's model.

**(1)**




a) $P(10 < T < 30) = P(A) + P(B) + P(C) + P(D)$

$$= \frac{(2 \times 4.2) + (4 \times 4) + (4 \times 3.5) + (16 \times 1)}{90} \quad (1)$$

$$= \frac{8.4 + 16 + 14 + 10}{90}$$

$$= \frac{48.4}{90}$$

$$= 0.5377\ldots = 0.54 \text{ (2 s.f.)} \quad (1)$$

(b) It does not look suitable because a normal distribution is symmetrical and the histogram is not. (1)

c) $u = x \qquad v = -e^{-x}$

$u' = 1 \qquad v' = e^{-x}$ (1)

$u \times v - \int vu'$

$\therefore \int xe^{-x}\,dx = -xe^{-x} - \int (-e^{-x})\,dx$ (1)

$= -xe^{-x} + \int e^{-x}\,dx$

$= -xe^{-x} - e^{-x}$

$\therefore \int_0^n xe^{-x}\,dx = \left[-xe^{-x} - e^{-x}\right]_0^n$

$= (-ne^{-n} - e^{-n}) - (-e^{0})$ (1)

$= -ne^{-n} - e^{-n} + 1$

$= 1 - (n+1)e^{-n}$ (1) (shown)

d) Area under frequency polygon = 90 when $n = 4$

$\therefore k \int xe^{-x}\,dx = 90$ (1)

$\therefore k\{1 - (n+1)e^{-n}\} = 90$

when $n = 4$ : $k(1 - 5e^{-4}) = 90$ (1)

$k = \dfrac{90}{1 - 5e^{-4}}$

$k = 99.07\ldots$ (1)

$\therefore k \approx 99$

e) (i) $T \sim N(14.9, 9.3^2)$

$P(10 < T < 30) = 0.6486\ldots \approx 0.649$ (3s.f.) 1

(ii) No° of patients

$= \int_1^3 99xe^{-x}\,dx$

$= 99\{(1-4e^{-3}) - (1-2e^{-1})\}$

$\approx 53.1\ldots$ 1

∴ Probability $= \dfrac{53.1\ldots}{90}$

$= 0.590\ldots$ 1

(f) Some patients might stay longer than 40 hours. 1